

VISUALISATION PROCESSES IN SELECTED NAMIBIAN MATHEMATICS CLASSROOMS

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ABSTRACT – This paper briefly reports on three recent visualization processes in mathematics education case studies that trialled visualization teaching approaches in different contexts within Namibia. All participating teachers were selected purposefully. The studies found that visualisation processes play an essential role in enhancing understanding of varied mathematical concepts at all levels of school education in these selected Namibian mathematics classes. We conclude that as much as visualization is concerned with physical images, products and diagrams, it is also about mental processes in solving problems.

Keywords: Visualisation; Mathematics; Visual Models; Gestures

INTRODUCTION

This paper briefly reports on three completed instrumental cases on visualization processes in three different grades/phases in the school curriculum on fractions, quadrilaterals and the use of gestures that were done in Namibia between January 2017 and October 2018. The studies were conducted under the auspices of the Visualization Processes in mathematics education (VIPROmaths) research project that aims to research the effective use of visualisation processes in mathematics classrooms in South Africa, Namibia, Zambia, Switzerland and Germany. This paper wishes to advocate for the VIPROmaths project that has at its core a research programme that interrogates diverse aspects of visualization processes in the context of teaching and learning mathematics. Overall, the VIPROmaths project (www.ru.ac.za/mathsedchair and www.nammaths.com) is framed by research agendas that fall into four main clusters; visualization as a pedagogy, visualization and cognition, visualization as a key mathematical construct, and visualization as a medium for epistemological access. Currently it involves a total of fourteen MEd and seven PhD students from South Africa and Namibia. Two of the three studies briefly outlined here were all crafted around a dedicated teacher intervention program. It was the aim of each study to generate new knowledge in and insights into strategically harnessing visualization processes in selected Namibian mathematics classrooms to make a positive contribution to improving classroom practice and pedagogy in Namibia. The newly revised mathematics curriculum in Namibia intimates that mathematics should be made visual to all learners (Namibia: Ministry of Education (MoE), 2015)

As the primary goal of schooling is the provision of meaningful learning environments, the teachers' primary task is thus to find and use teaching approaches that promote conceptual understanding of mathematical concepts, ideas and relationships. This has implications for every teacher in terms of how and what he/she teaches. Although the Namibian national curriculum for basic education compels mathematics teachers to be "creative and innovative to produce their own teaching and learning materials linked to practice" (Namibia. Ministry of Education (MoE), 2010, p. 6) at all school levels, most mathematics teachers still find it difficult to create and make use of teaching aids and materials. Miranda and Adler (2010) observe that "Namibia is one of the many African countries, in which the use of manipulatives in mathematics classrooms is not a common practice" (p. 17). This is corroborated by our experiences that confirm that apart from diagrams and figures encountered in the prescribed textbooks (Nghifimule, 2017); meaningful visualization processes are sparsely used in Namibian mathematics classrooms. It is thus the objective of the various intervention programs central to the VIPROmaths project to encourage increased and sustained use of visualization processes to enhance conceptual understanding.

VISUALISATION

Visualisation is generally accepted and considered as helpful in mathematics education because of its diverse pedagogic, cognitive and epistemic purposes. We consider Arcavi's

(2003) definition of visualisation particularly helpful as it draws together various aspects (such as processes and products) of visualization in the mathematics classroom:

Visualization is the ability, the process and the product of creation, interpretation, use of, and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings.' (p. 217).

Arcavi (2003, pp. 216 - 7) describes mathematics as a human and cultural creation that deals with objects and entities quite different from physical phenomena and relies heavily on visualisation in its different forms and at different levels, far beyond the obviously visual field of geometry, and spatial visualisation. We thus concur with Zimmerman and Cunningham (1991) who noted that visualisation in mathematics is not merely an appreciation of mathematics through pictures or visuals, but instead it supplies depth and meaning to understanding mathematics, serving as a reliable guide to problem solving, and inspiring creative discoveries.

Visual representations play a crucial role in understanding and making sense of mathematics. Some of the ways in which visualisation processes in mathematics education have been used include making use of one's own visual images when posing and solving mathematics problems (Bishop, 1989); constructing concept maps as visuals (Presmeg, 2006); using visualisation as a teaching strategy; and designing "visualities" of mathematical tasks themselves. Boaler (2016, p. 1) argues, "when students learn through visual approaches, mathematics changes for them, and they are given access to deep and new understandings."

It is expected from an effective teacher that she develops and employs appropriate visual materials or use ready-made materials such as teaching aids and manipulatives in order to enrich her teaching. As stated by Boaler (2016, p. 5), "teachers who emphasize visual mathematics and who use well-chosen manipulatives encourage higher achievement for students, not only in elementary school..." Visualisation in mathematics pedagogy thus refers to the deliberate use of visuals to promote a deep understanding of concepts "both from teachers introducing mathematical ideas visually, and students using visuals to think and make sense of mathematics and connecting previously unconnected theories in mathematics," (Boaler, 2016, p. 5, 7). Rösken and Rolka (2006) assert that visualisation can be a powerful tool to explore mathematical problems and to give meaning to mathematical concepts and the relationship between them, and subsequently reduce complexity when dealing with a multitude of information.

This paper very briefly reports on three recent case studies that trialled visualization teaching approaches in different contexts within the framing of three intervention programmes. Except for Nelao's intervention programme, each of the other two consisted of a set of workshops that firstly orientated the selected participants in visualization processes in mathematics, secondly in integrating these processes in the teaching of fractions, quadrilaterals and basic geometry respectively, and thirdly in designing and implementing a series of lessons in these mathematical concepts. All the participants were selected purposefully and were located in close proximity to the researchers.

AUNE'S PROJECT AND FINDINGS - FRACTIONS

This study, guided by constructivist theory, explored the use of three visual models (area model, number line model, set model) by three selected secondary school mathematics teachers in the Khomas region in their teaching of common fractions to enhance the conceptual understanding. The overarching aim of this study was to create awareness amongst mathematics education teachers about the role of visualisation processes in the teaching and learning of common fractions and to trial the three visual models. This mixed method study firstly employed a survey questionnaire with all mathematics teachers in the region (forty-six teachers) to seek responses on the extent to which the respondents used visual models when teaching fractions. Secondly, the study used stimulated recall interviews

with the three selected Grade 8 teachers on how they used the three visual models in their day-to-day teaching of common fractions because of participating in an intervention programme that interrogated the three visual fraction models.

Although 80% (n=46) of the teachers in the survey indicated that they rarely used visual models to teach fractions, many agreed that the use of visual models such as area, number line or set models to teach common fractions was important. The 76% of the surveyed teachers indicated that visual models can make abstract concepts concrete and can clarify mathematical ideas whose meanings are difficult to comprehend. In addition, teachers in the survey further asserted that visual models attract learners' attention and stimulate learners' interest in learning mathematics. They further argued that visual models also make mathematics fun and practical. Visual models enhance deep conceptual understanding. They highlighted that visual models help learners to grasp mathematical concepts without difficulty as they learn better by seeing visuals rather than symbols.

It was noted that despite interrogating all three fraction models in the intervention programme, the three participating teachers were reluctant to use the set model. This was followed up in the interviews. Some of the teachers' responses are given below.

Ms Nalo had this to say:

Ok, to me both models help learners to understand, only that each one of them has a kind of a challenge but they are both easier to use than the set model. I used only two models, the area and number line only in my teaching. I tried the set model and realised that it is difficult. But the area model is quite easy because I could connect easily with other concepts. Same with the number line I could also link with number sense but the set model is not easy to understand. (NaSRI₁ 96-104)

On the same note, Mr Malele pointed out that:

...the two models that I have used, they are both helpful any way. But I have seen that using area models is quite easier than the other two. The number line when you draw it you have to divide the number line into continuous whole which is sometimes in my teachings made learners a bit confused. But when I have used the area model, where you just draw separate wholes and you divide them into parts as given by the fractions that makes learners to understand it much better compared to the number line model. The set model I did not use it. I found it difficult to use with my learners. I would need more time to study it first. (MaSRI₃ 123-127).

All three teachers used only the area and the number line models, and not the set model, to teach their lessons on fractions that they prepared in the intervention programme. These were mostly drawn on the chalkboard except for one teacher who used PowerPoint to present the addition and subtraction of fractions using the number line model. Two teachers preferred drawing the models while their learners were present in class, and one preferred to draw the models before learners came to class. Observations and feedback from the teachers revealed that learners tended to follow and understand better when models were drawn during the lesson, unlike when they find models already drawn. This was because the drawing was always accompanied by teacher explanations. This has implications on the use of already made visuals like charts, PowerPoint presentation slides and even textbook use when teaching mathematics.

Mr Mose, for example indicated that:

...the area model is easy and convenient to use. With the area model, we were cutting into smaller pieces, of equal parts and of a different colour. The area model is the best and it is user friendly. Learners were able to draw and play around with it unlike with the number line and set model. The number is also easier but I prefer the area model. The other one, the set model, aaah I avoided it. It is confusing. (MoSRI₁ 37-41).

The dominance of the area model in the observed classes was due to this model being easier to draw and use, and the ability to link the area model to prior knowledge. Ms Nalo argued that the number line model was not as easy to use as the area model when adding or subtracting fractions of different denominators. On the other hand, Mr Malele pointed out that a number line is confusing because of the continuous whole. All the three teachers avoided the set model. They considered it very difficult for them and the learners to understand.

GIVEN'S PROJECT AND FINDINGS - QUADRILATERALS

This study aimed at investigating and analysing the effective use of a Geoboard by three selected teachers as a visualisation tool in the teaching of quadrilaterals in Grade 7 as a result of them participating in an intervention programme in the Kunene Region. The study was also guided by the constructivist theory. Qualitative data was collected through observations and interviews from the three Grade 7 mathematics teachers in three primary schools.

Selected teachers participated first, in two workshops that aimed at designing their own Geoboards. The second set of three 1-hour workshops were aimed at orienting and familiarising teachers on how Geoboards may be used to construct and manipulate various types of plane shapes. They were also used to design a learning programme that consisted of six lesson plans. The teachers found the Geoboard useful in enhancing the teaching and understanding of properties of quadrilaterals. It served as a powerful visualization tool to illustrate, explore and discuss the properties of quadrilaterals. Its use aided motivation and improved participation during teaching. The teachers indicated that the Geoboard enabled a dynamic learning environment as learners were noted turning their boards in various directions and orientations to understand the properties of quadrilaterals they had constructed.

Mr Jones indicated that:

With the Geoboard it was very easy because you just take the rubber bands and you can adjust them even learners could adjust them the way they feel like. I mean extend the corners of a square or to put them inside to form a rhombus. (JoSRI₂ 56-59)

Ms Ruth noted that her:

The learners turned around the geoboard and then counted the pins at the now bottom. Instead of four pins now she had six, of the same trapezium. Although only the position was turned around it remained a trapezium yet the learner looked at it from a different angle. Unlike the way it was placed on the Geoboard. (RuSRI₃ 44-57)

Ms Smith pointed out that:

Learners were sharing the Geoboards in groups and every one was interested in doing something, in constructing something, everybody wanted to touch here and there. Therefore, everybody was actively participating there although not all the members of the group knew exactly what they were constructing. They were so eager to do and finish the individual activity. Everybody was eager to do it on his or her own (SmSRI₁ 93-97)

Despite the overwhelming positive reaction to using the Geoboard, numerous interesting challenges had to be overcome. The teachers found that the size of the Geoboard as a teaching aid had to be considered. It was important that the different coloured elastic bands that were used to construct the quadrilaterals were visible to the entire class.

Some geometric concepts were not so easy and apparent to demonstrate on the Geoboard. Mr Jones gave an example of lines of symmetry. Learners had difficulties constructing a line of symmetry when they constructed a square that had sides composed of an even number of pins.

Mr Jones, for example indicated:

What I realised is that if you are making a square using rubber bands on the Geoboard you must be very careful with the pins. Because the pins should be equal so that when

they use the rubber bands to make the line of symmetry, they must put it on the middle pins. So, if it is not like that, it becomes a problem because it won't give the exact properties (JoSRI₂, 147-153).

Mr Jones thus suggested that using pins and rubber bands for illustrating the concept of lines of symmetry on the Geoboard would work well when an odd number of pins are used. This is because of the existence of a clear set of pins in the middle. He noted that this was not the case when an even number of pins on the sides of a shape were used. Learners' failure to locate a line of symmetry on the Geoboard may have been caused by teachers. It seems they did not take into consideration the issue of even and odd numbers of pins on the Geoboard before giving the activity of symmetry to learners. Sarama and Clements' (2016) advice when it comes to the use of manipulatives, teachers should make sure that there is a connection between the manipulative and the concept related to its representation. However, it seems this was not heeded to as the anticipation of the problem that could arise regarding the use of an even number of pins was overlooked.

NELAO'S PROJECT AND FINDINGS - GESTURES

The third study was guided by the observation that when teachers talk and teach, they very often gesture, and those gestures often reveal information that may not be apparent in their verbal speech. In this study, gestures refer to bodily movements, usually with our fingers, hands and arms that accompany a verbal explanation or statement that were used to support, exemplify or illustrate what teachers were saying. This qualitative interpretive study framed by an enactivist perspective aimed to investigate the nature and role of gestures that three purposeful selected junior primary phase (Grades 0-3) teachers used in the teaching of mathematics. The study also aimed at understanding the selected mathematics teachers' views on the roles of their gestures as visualisation tools in the teaching of mathematics. Data was collected through classroom video-recorded observation and stimulated recall interviews.

Nelao's findings were based on 30 lesson observations across an entire term involving three foundation phase teachers. Her analytical framework, which she imposed on her video-recorded lessons, was grounded in McNeill's (1992) classification of gestures which included pointing (deictic) gestures, iconic (illustrative) gestures, metaphoric gestures, beat (motor) gestures and symbolic (emblem) gestures. This study aligned itself with Castellon and Enyedy's (2006) argument that gestures can be important visual resources that can play a valuable role in the teaching-learning process of mathematics. Very pertinently, they can be used as an important bridge between imagery and speech. They may be seen as a nexus bringing together action, memory, speech, imagery and mathematical problem solving (Alibali & Nathan, 2012).

The analysis was two pronged where firstly a frequency analysis was done across all the participating teachers for all the 30 lessons – see table 1, and secondly a qualitative analysis of the interviews that were conducted with each of the teachers.

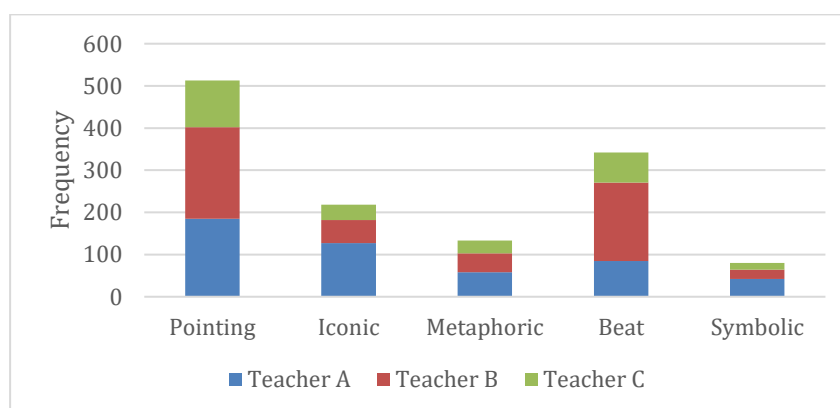


Figure 1: Frequency of gestures used

In general, the participating teachers felt that the use of gestures provided for a learning environment that was dynamic and rich. Gestures enabled the teachers to reinforce, support, illustrate and strengthen concepts they were trying to teach. Gestures facilitated both instructional and conceptual communication, the former being communication that refers to procedures of what needs to be done, and the latter being communication that is mathematical referring to concepts and content. The study concluded by acknowledging that gestures are recognized as legitimate teaching resources and strategies, provided they are used strategically and meaningfully.

CONCLUSION

It seems as if we take it for granted that visuals are integral to our interpersonal and teaching communication repertoire. Visuals are closely linked with language and thought, and therefore with teaching and learning. There is increased recognition that visualisation is not meant for illustrative purposes only, but is a key component of reasoning and mathematical thinking. As much as visualization is concerned with physical images, products and diagrams, it is also about mental processes in solving problems. As the three studies above illustrated, there is room for much research about how best to harness visualization processes in all its facets in the teaching and learning of mathematics.

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